

# Brane World Cosmologies with Varying Speed of Light

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## Abstract

We study cosmologies in the Randall-Sundrum models, incorporating the possibility of time-varying speed of light and Newton's constant. The cosmologies with varying speed of light (VSL) were proposed by Moffat and by Albrecht and Magueijo as an alternative to inflation for solving the cosmological problems. We consider the case in which the speed of light varies with time after the radion or the scale of the extra dimension has been stabilized. We elaborate on the conditions under which the flatness problem and the cosmological constant problem can be resolved. We find that the RS models are more restrictive about possible desirable VSL cosmological models than the standard general relativity. Particularly, the VSL cosmologies may provide with a possible mechanism for bringing the quantum corrections to the fine-tuned brane tensions after the SUSY breaking under control.

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# 1 Introduction

Although successful in passing some crucial observational tests, the Standard Big Bang (SBB) model is regarded as being incomplete in the sense that it does not address the state of universe at initial period and many features of the SBB model are just assumed as initial conditions. As a consequence, the SBB model has many cosmological problems which the SBB model can avoid only by assuming unnatural or fine-tuned initial conditions. The inflationary scenario [1, 2, 3] was introduced in an attempt to complete the SBB picture as its precursor and solve some of the cosmological problems. The inflationary scenario assumes a period of superluminary or accelerated expansion during an initial period (prior to a conventional non-inflationary evolution described by the SBB model) due to a scalar field (inflaton) with an inflationary potential satisfying the slow-roll approximation. The inflationary models successfully solve the horizon problem, the flatness problem, the homogeneity problem, and the problem with a variety of unwanted relics without assuming unnatural initial conditions. The inflationary models also have been favored as prominent candidates for explaining the origin of structure in the universe.

The varying speed of light (VSL) theory was proposed [4, 5] as an alternative to inflation for solving the cosmological problems. Instead of introducing an inflaton to make the universe undergo superluminal expansion while violating the strong energy condition, the VSL models just assume without changing the universe matter content that the speed of light initially took a larger value and then decreased to the present value at an early time, thereby violating the Lorentz invariance. As the speed of light is assumed to have taken a larger value at an early time, the horizon problem is automatically solved. The VSL models also solve [4, 5, 6, 7, 8, 9, 10] other various cosmological problems that the inflationary models solve. What is appealing about the VSL models is that unlike the inflationary models the cosmological constant problem can be solved if the rapid enough decreasing speed of light is assumed. The claim in Ref. [11] of the experimental evidence for a time-varying fine structure constant  $\alpha = e^2/(4\pi\hbar c)$  suggests that the speed of light may indeed vary with time. (Other possibility of time-varying electric charge  $e$  was considered in Ref. [12].) However, in terms of theoretical foundation, the VSL models are not yet as well developed as the inflationary ones. (Some of attempts can be found in Refs. [4, 13, 14, 15, 16, 17, 18, 19].)

In the brane world scenario with warped compactification [20, 21, 22], it is shown that the light signal can travel faster through the extra dimension and thereby the horizon problem can be solved [23, 24, 25, 26]. Further aspect of the Lorentz violation, which is a necessary requirement (Cf. Ref. [27]) for the VSL models, was explored in Ref. [28, 29], where the possibility of gravitons, instead of photons, traveling faster is considered. Ref. [30] gave a toy example on the brane world cosmology where the speed of light varies with time. (See also Ref. [31] for further study on this aspect.)

So, the brane world scenario may provide with a natural framework or mechanism for realizing the VSL cosmology. Moreover, since the VSL models solve the cosmological constant problem, brane world cosmologies with variable speed of light may be used to control the quantum corrections to the fine-tuned brane tensions after the SUSY breaking, just like the mechanism for self-tuning brane tension [32, 33]: While the speed of light changes with time, the quantum corrections to the brane tensions get converted into ordinary matter, thereby the brane tensions being pushed back to the fine-tuned values.

In this paper, we study the VSL cosmologies in the Randall-Sundrum (RS) models. We consider the case in which the speed of light changes with time after the radion or the scale of the extra dimension has been stabilized. Following the previous works [4, 5, 6], we just assume the speed of light to change either suddenly at some critical time or gradually as a power law in the cosmic scale factor without specifying the mechanism for generating time-variable speed of light, for the purpose of seeing consequences for varying speed of light in the brane world cosmologies. It is a subject of future research to find natural mechanism for realizing the time-varying speed of light within realistic brane world cosmologies. We elaborate on conditions under which the flatness problem and the cosmological constant problem can be resolved.

The paper is organized as follows. In section 2, we study the VSL cosmology in the RS model with one positive tension and one negative tension branes (RS1 model). We consider the cases of sudden and gradual changes of the speed of light with time. In section 3, we study the VSL cosmology in the RS model with one positive tension brane (RS2 model) for the case when the speed of light suddenly changes with time.

## 2 The VSL Cosmology in the RS1 Model

In this section, we study the cosmology in the RS model with one positive tension and one negative tension branes, considering the possibility of time-varying speed of light and five-dimensional Newton's constant. We place the brane with positive tension  $\sigma_0$  [negative tension  $\sigma_{1/2}$ ] at  $y = 0$  [at  $y = 1/2$ ]. The bulk contains a cosmological constant  $\Lambda$ . We add a massive bulk scalar  $\Phi$  with potentials  $V_{0,1/2}(\Phi)$  on the branes at  $y = 0, 1/2$ , having nontrivial VEV, to stabilize the radion through the Goldberger-Wise mechanism [34, 35, 36]. The expansion of the brane universe is due to matter fields with the Lagrangian densities  $\mathcal{L}_{0,1/2}$  on the branes at  $y = 0, 1/2$ .

We assume that the speed of light  $c$  varies with time during some initial period of cosmological evolution.  $c$  may go through a first order phase transition at a critical time  $t_c$ , suddenly jumping from some superluminal value  $c_-$  to the present value  $c_+$  of the speed of light, as was considered in Refs. [4, 5], or may change gradually like  $c(t) = c_0 a^n$  for some constants  $c_0$  and  $n$  [6]. In Ref. [4], it is proposed that the

superluminary phase of the former case may be due to the spontaneous Higgs breaking of local Lorentz invariance from  $SO(3, 1)$  to  $SO(3)$  above some critical temperature  $T_c$ . In general, if  $c$  is variable, then the Lorentz invariance becomes explicitly broken. It is postulated in VSL models that there exists a preferred Lorentz frame in which the laws of physics simplify. In such preferred frame, the action is assumed to be given by the standard (Lorentz invariant) action with a constant  $c$  replaced by a field  $c(x^\mu)$ , *the principle of minimal coupling*. However, such simplified laws of physics are not invariant under the frame transformation.

So, the action under consideration is given by

$$S = \int d^5x \left[ \sqrt{-\hat{g}} \left( \frac{\psi}{16\pi G_5} \mathcal{R} - \Lambda - \frac{1}{2} \partial_M \Phi \partial^M \Phi + \frac{1}{2} m^2 \Phi^2 \right) + \mathcal{L}_\psi \right] \\ + \int d^4x \sqrt{-g} [\mathcal{L}_0 + V_0(\Phi)|_0 - \sigma_0] + \int d^4x \sqrt{-g} [\mathcal{L}_{1/2} + V_{1/2}(\Phi)|_{1/2} - \sigma_{1/2}], \quad (1)$$

where in analogy with the Brans-Dicke theory we defined a scalar field  $\psi \equiv c^4$  out of varying speed of light  $c$ ,  $G_5$  is the five-dimensional Newton's constant<sup>2</sup>, and  $\mathcal{L}_\psi$  controls the dynamics of  $\psi$ . It is required that  $\mathcal{L}_\psi$  should be explicitly independent of the other fields, including the metric, so that the principle of minimal coupling continues to hold for the field equations. This requirement implies that Bianchi identities  $\mathcal{G}^{\mu\nu}{}_{;\nu} = 0$  of the Einstein tensor  $\mathcal{G}_{\mu\nu}$  continue to hold and thereby the energy-momentum conservation law is modified by an additional term proportional to  $\dot{c}/c$ . (Cf. See the Einstein's equations (4) in the below.) This violation of the energy-momentum tensor conservation during the phase of varying speed of light could provide with an explanation for the creation of matter in the beginning of universe. This matter creation process is somehow analogous to the reheating process of the inflationary models, by which the inflaton's energy density is converted to conventional matter after inflation.

As usual, the metric Ansatz for the expanding brane universe is given by

$$\hat{g}_{MN} dx^M dx^N = -n^2(t, y) c^2 dt^2 + a^2(t, y) \gamma_{ij} dx^i dx^j + b^2(t, y) dy^2, \quad (2)$$

where  $\gamma_{ij}$  is the metric for the maximally symmetric three-dimensional space given in the Cartesian and the spherical coordinates by

$$\gamma_{ij} dx^i dx^j = \left( 1 + \frac{k}{4} \delta_{mn} x^m x^n \right)^{-2} \delta_{ij} dx^i dx^j = \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (3)$$

with  $k = -1, 0, 1$  respectively for the three-dimensional space with the negative, zero and positive spatial curvature.

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<sup>2</sup>With our convention of the D-dimensional Newton's constant  $G_D$ , i.e.  $\kappa_D = 8\pi G_D/c^4$ , which most of papers follow, the Newton's law of gravitation, which is reproduced in the weak gravitational field and low velocity limit of the Einstein's gravity, takes the form  $F_{12} = \frac{4\pi G_D}{\Omega_{D-2}} \frac{m_1 m_2}{r^{D-2}}$ , where  $\Omega_{D-2}$  is the volume of the unit  $(D-2)$ -sphere, instead of the standard form  $F_{12} = G_D \frac{m_1 m_2}{r^{D-2}}$ .

Since  $\psi$  is minimally coupled, the Einstein's equations take the conventional form:

$$\mathcal{G}_{MN} = \frac{8\pi G_5}{\psi} \mathcal{T}_{MN}, \quad (4)$$

with the energy-momentum tensor given by

$$\begin{aligned} \mathcal{T}_{MN} = & \partial_M \Phi \partial_N \Phi - \frac{1}{2} \hat{g}_{MN} \partial_P \Phi \partial^P \Phi + \hat{g}_{MN} \left( \frac{1}{2} m^2 \Phi^2 - \Lambda \right) \\ & + \delta_M^\mu \delta_N^\nu \left[ \frac{\delta(y)}{b} \left\{ \mathcal{T}_{\mu\nu}^0 + (V_0(\Phi) - \sigma_0) g_{\mu\nu} \right\} \right. \\ & \left. + \frac{\delta(y-1/2)}{b} \left\{ \mathcal{T}_{\mu\nu}^{1/2} + (V_{1/2}(\Phi) - \sigma_{1/2}) g_{\mu\nu} \right\} \right], \end{aligned} \quad (5)$$

where the energy-momentum tensors  $\mathcal{T}_{\mu\nu}^{0,1/2} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_{0,1/2})}{\delta g^{\mu\nu}}$  for matter fields on the branes have the following usual perfect-fluid forms:

$$\mathcal{T}^{0\mu}_\nu = \text{diag}(-\varrho_0 c^2, \wp_0, \wp_0, \wp_0), \quad \mathcal{T}^{1/2\mu}_\nu = \text{diag}(-\varrho_{1/2} c^2, \wp_{1/2}, \wp_{1/2}, \wp_{1/2}). \quad (6)$$

As proposed in Ref.[5], the time-variable  $c$  does not introduce corrections to the Einstein tensor  $\mathcal{G}_{MN}$  for the bulk metric (2) in the preferred frame:

$$\begin{aligned} \mathcal{G}_{00} &= 3 \left[ \frac{1}{c^2} \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) - \frac{n^2}{b^2} \left\{ \frac{a'}{a} \left( \frac{a'}{a} - \frac{b'}{b} \right) + \frac{a''}{a} \right\} + k \frac{n^2}{a^2} \right], \\ \mathcal{G}_{ij} &= \frac{a^2}{b^2} \left[ \frac{a'}{a} \left( 2 \frac{n'}{n} + \frac{a'}{a} \right) - \frac{b'}{b} \left( \frac{n'}{n} + 2 \frac{a'}{a} \right) + 2 \frac{a''}{a} + \frac{n''}{n} \right] \gamma_{ij} \\ &+ \frac{a^2}{c^2 n^2} \left[ \frac{\dot{a}}{a} \left( 2 \frac{\dot{n}}{n} - \frac{\dot{a}}{a} \right) + \frac{\dot{b}}{b} \left( \frac{\dot{n}}{n} - 2 \frac{\dot{a}}{a} \right) - 2 \frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} \right] \gamma_{ij} - k \gamma_{ij}, \\ \mathcal{G}_{04} &= \frac{3}{c} \left[ \frac{n' \dot{a}}{n a} + \frac{a' \dot{b}}{a b} - \frac{\dot{a}'}{a} \right], \\ \mathcal{G}_{44} &= 3 \left[ \frac{a'}{a} \left( \frac{a'}{a} + \frac{n'}{n} \right) - \frac{b^2}{c^2 n^2} \left\{ \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) + \frac{\ddot{a}}{a} \right\} - k \frac{b^2}{a^2} \right], \end{aligned} \quad (7)$$

where the overdot and the prime respectively denote derivatives w.r.t.  $t$  and  $y$ . Due to the assumption of minimal coupling, the field equation for  $\Phi$  also takes the usual form:

$$\begin{aligned} c^{-1} \partial_t \left( n^{-1} a^3 b c^{-1} \partial_t \Phi \right) - \partial_y \left( n a^3 b^{-1} \partial_y \Phi \right) \\ - n a^3 b \left[ m^2 \Phi + V'_0 \frac{\delta(y)}{b} + V'_{1/2} \frac{\delta(y-1/2)}{b} \right] = 0, \end{aligned} \quad (8)$$

where the primes in the potentials denote derivatives w.r.t.  $\Phi$ .

When there is no matter field on the branes, the equations of motion admit the static brane solution [20] with the metric components given by

$$n(y) = a(y) = e^{-m_0 b_0 |y|}, \quad b(y) = b_0 = \text{const.} \quad (9)$$

The brane tensions take the fine-tuned values given by

$$\sigma_0 = \frac{3c^4 m_0 b_0}{4\pi G_5} = -\sigma_{1/2}, \quad \Lambda = -\frac{3c^4 m_0^2 b_0^2}{4\pi G_5}, \quad (10)$$

from which we see that  $m_0$  varies with  $t$ , if  $c$  varies with  $t$ . So, the solution at  $y \neq 0$  is not static when  $\dot{c} \neq 0$ , if the the brane tensions  $\sigma_{0,1/2}$  are constant, since the parameter  $m_0$  has to vary with time to keep  $\sigma_{0,1/2}$  constant. In order for the solution (9) to remain static (i.e., for  $m_0$  to remain constant) while  $c$  varies with  $t$ , the fine-tuned brane tensions  $\sigma_{0,1/2}$  have to vary with  $t$ , as specified by Eq. (10).

When matter fields are included on the branes, the brane universe undergoes cosmological expansion. In Refs. [37, 38], by expanding the metric around the static solution (9) to the linear order in  $\varrho_{0,1/2}$  and then taking average over  $y$ , the following effective Friedmann equations for the expanding brane universe is obtained:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_4}{3} \left(\varrho_0 + \varrho_{1/2}\Omega_0^4\right) - \frac{kc^2}{a^2}, \quad (11)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_4}{3} \left[\varrho_0 + \varrho_{1/2}\Omega_0^4 + \frac{3}{c^2} \left(\wp_0 + \wp_{1/2}\Omega_0^4\right)\right], \quad (12)$$

where

$$\Omega_0 \equiv e^{-m_0 b_0/2}, \quad G_4 = \frac{m_0 G_5}{1 - \Omega_0^2}. \quad (13)$$

Note,  $G_4$  also varies with  $t$ , because of the dependence of  $m_0$  and  $\Omega_0$  on  $c$ . From the above effective Friedmann equations, we obtain the following generalized conservation equation:

$$\dot{\varrho} + 3\left(\frac{\wp}{c^2} + \varrho\right)\frac{\dot{a}}{a} = -\varrho\frac{\dot{G}_4}{G_4} + \frac{3kc\dot{c}}{4\pi G_4 a^2}, \quad (14)$$

where

$$\varrho \equiv \varrho_0 + \varrho_{1/2}\Omega_0^4, \quad \wp \equiv \wp_0 + \wp_{1/2}\Omega_0^4. \quad (15)$$

So, while  $c$  and/or  $G_4$  vary with time, mass is not conserved, namely that matter is created or taken out of the brane universe.

In section 2.1 and 2.2, we study the conditions under which the flatness problem and the cosmological constant problem can be resolved by assuming suddenly changing  $c$  at some critical time. In section 2.3, we consider the case in which  $c$  gradually changes as a power law in the scale factor  $a$ .

## 2.1 The flatness problem

In this subsection, we illustrate how the flatness problem can be resolved by assuming the time-varying  $c$ . As the Friedmann equations (11,12) for the brane universe have

the same form as those of the standard cosmology, the discussion is along the same line as Ref. [6]. The critical density  $\varrho_c$ , the mass density corresponding to  $k = 0$  for a given  $\dot{a}/a$ , of the brane universe is given by

$$\varrho_c = \frac{3}{8\pi G_4} \left( \frac{\dot{a}}{a} \right)^2. \quad (16)$$

We define the deviation of  $\varrho$  from  $\varrho_c$  as  $\epsilon \equiv \varrho/\varrho_c - 1$ . So,  $\epsilon < 0$ ,  $\epsilon = 0$ , and  $\epsilon > 0$  cases respectively correspond to the open ( $k = -1$ ), flat ( $k = 0$ ) and closed ( $k = 1$ ) universes. Differentiating  $\epsilon$  w.r.t.  $t$ , we have

$$\dot{\epsilon} = (1 + \epsilon) \left( \frac{\dot{\varrho}}{\varrho} - \frac{\dot{\varrho}_c}{\varrho_c} \right). \quad (17)$$

We assume that the brane matter satisfies the equation of state of the form:

$$\wp = w\varrho c^2, \quad (18)$$

with a constant  $w$ . Then, making use of Eqs. (11,12,14), we have

$$\begin{aligned} \frac{\dot{\varrho}}{\varrho} &= -3\frac{\dot{a}}{a}(1+w) - \frac{\dot{G}_4}{G_4} + 2\frac{\dot{c}}{c}\frac{\epsilon}{1+\epsilon}, \\ \frac{\dot{\varrho}_c}{\varrho_c} &= -\frac{\dot{a}}{a}[2 + (1+\epsilon)(1+3w)] - \frac{\dot{G}_4}{G_4}. \end{aligned} \quad (19)$$

Substituting these into Eq. (17), we have

$$\dot{\epsilon} = (1 + \epsilon)\epsilon\frac{\dot{a}}{a}(1 + 3w) + 2\frac{\dot{c}}{c}\epsilon. \quad (20)$$

So, during the SBB evolution (with  $\dot{c} = 0$ ), any ordinary matter field satisfying the strong energy condition  $1 + 3w > 0$  drives  $\epsilon$  away from zero. The fact that the value of  $\epsilon$  observed in our present universe is close to zero (Cf. see Refs. [39, 40, 41] for recent observational data) indicates that  $\varrho$  at early universe has to be remarkably close to  $\varrho_c$  to avoid too much deviation from  $\varrho_c$  at present time, the so-called flatness problem of the SBB model. The inflationary cosmology solves the flatness problems, as can be seen from Eq. (20): Since the inflaton scalar violates the strong energy condition,  $\epsilon$  is driven towards zero during the inflationary era. The VSL models can also solve the flatness problem, if  $c$  decreases rapid enough: If  $|\dot{c}/c| \gg \dot{a}/a$ , then the first term on the RHS of Eq. (20) can be neglected. So, while  $c$  decreases,  $\epsilon$  is rapidly driven to zero. Just like the inflationary models, only the flat universe ( $k = 0$ ) is stable while  $c$  decreases.

## 2.2 The cosmological constant problem

We begin by illustrating the cosmological constant problem in the standard cosmology and its resolution by the VSL model. When the contribution  $\varrho_\Lambda = \Lambda c^2/(8\pi G_4)$  of the cosmological constant  $\Lambda$  is included in  $\varrho$ , i.e.  $\varrho = \varrho_m + \varrho_\Lambda$  where  $\varrho_m$  is the mass density of normal matter, then the generalized conservation equation (14) is modified to

$$\dot{\varrho}_m + 3\frac{\dot{a}}{a}\left(\varrho_m + \frac{\varrho_m}{c^2}\right) = -\dot{\varrho}_\Lambda - \varrho\frac{\dot{G}_4}{G_4} + \frac{3kc\dot{c}}{4\pi G_4 a^2}. \quad (21)$$

If we define  $\epsilon_\Lambda \equiv \varrho_\Lambda/\varrho_m$ , then its derivative w.r.t.  $t$  is

$$\dot{\epsilon}_\Lambda = \epsilon_\Lambda \left( \frac{\dot{\varrho}_\Lambda}{\varrho_\Lambda} - \frac{\dot{\varrho}_m}{\varrho_m} \right). \quad (22)$$

From the definition of  $\varrho_\Lambda$ , we have

$$\frac{\dot{\varrho}_\Lambda}{\varrho_\Lambda} = 2\frac{\dot{c}}{c} - \frac{\dot{G}_4}{G_4}, \quad (23)$$

and from Eq. (21), along with the Friedmann equations, we obtain

$$\frac{\dot{\varrho}_m}{\varrho_m} = -3\frac{\dot{a}}{a}(1+w) - 2\frac{\dot{c}}{c}\frac{\varrho_c}{\varrho_m} + 2\frac{\dot{c}}{c} - \frac{\dot{G}_4}{G_4}. \quad (24)$$

So, Eq. (22) takes the form:

$$\dot{\epsilon}_\Lambda = \epsilon_\Lambda \left[ 3\frac{\dot{a}}{a}(1+w) + 2\frac{\dot{c}}{c}\frac{1+\epsilon_\Lambda}{1+\epsilon} \right]. \quad (25)$$

Thus, in the SBB model (with  $\dot{c} = 0$ ), any matter satisfying  $w > -1$  drives  $\epsilon_\Lambda$  away from zero as the universe expands, leading to the value of the cosmological constant many orders of magnitude larger than the presently observed value in our universe. So, during the initial period of the cosmological evolution,  $\epsilon_\Lambda$  had to be fine-tuned to take extremely small value, the so called cosmological constant problem in cosmology. (To make matters worse, theoretical estimates of various contributions rather lead to the cosmological constant of the order of the Planck scale.) So, it might help, if there exist some mechanisms that drive  $\epsilon_\Lambda$  towards zero. The inflationary models cannot solve the problem, because the violation of the strong energy condition is not enough to make  $w+1$  negative. However, the VSL model can solve the cosmological constant problem, if  $|\dot{c}/c| \gg \dot{a}/a$ . Then, we can neglect the first term on the RHS of Eq. (25), so  $\epsilon_\Lambda$  is rapidly driven to zero while  $c$  decreases to the present value of the speed of light. In this process, vacuum energy density is discharged into ordinary matter.

We now consider the case of the brane world cosmology. Unlike the case of the standard cosmology, the mass density satisfying  $\varrho = -\wp/c^2$  is not directly related



to the cosmological constant of the brane universe, but rather to the brane tension. However, if we assume the brane tensions to have initially taken the fine-tuned values<sup>3</sup> (10) due to, for example, the supersymmetry embedding of the brane world scenario, then by identifying the nonzero mass density  $\varrho_{\delta\sigma}$  satisfying  $\varrho_{\delta\sigma} = -\wp_{\delta\sigma}/c^2$  as being due to the (quantum) correction  $\delta\sigma$  to the brane tensions (after the SUSY breaking) we can regard nonzero  $\varrho_{\delta\sigma}$  as the source of nonzero effective cosmological constant in the brane universe. The total mass density of the brane universe is then given by  $\varrho = \varrho_m + \varrho_{\delta\sigma}$ , where  $\varrho_m$  is the mass density of the ordinary matter on the brane and  $\varrho_{\delta\sigma} = \delta\sigma/c^2$ . So, the generalized conservation equation (14) is modified to

$$\dot{\varrho}_m + 3\frac{\dot{a}}{a}\left(\varrho_m + \frac{\wp_m}{c^2}\right) = -\dot{\varrho}_{\delta\sigma} - \varrho\frac{\dot{G}_4}{G_4} + \frac{3kc\dot{c}}{4\pi G_4 a^2}. \quad (26)$$

Defining  $\epsilon_{\delta\sigma} \equiv \varrho_{\delta\sigma}/\varrho_m$ , we obtain

$$\dot{\epsilon}_{\delta\sigma} = \epsilon_{\delta\sigma} \left( \frac{\dot{\varrho}_{\delta\sigma}}{\varrho_{\delta\sigma}} - \frac{\dot{\varrho}_m}{\varrho_m} \right). \quad (27)$$

By using the Friedmann equations, the generalized conservation equation and the definition of  $\epsilon_{\delta\sigma}$ , we obtain

$$\frac{\dot{\varrho}_{\delta\sigma}}{\varrho_{\delta\sigma}} = -2\frac{\dot{c}}{c}, \quad (28)$$

$$\frac{\dot{\varrho}_m}{\varrho_m} = -3\frac{\dot{a}}{a}(1+w) - 2\frac{\dot{c}}{c}\frac{\varrho_c}{\varrho_m} + 2\frac{\dot{c}}{c}\frac{\varrho + \varrho_{\delta\sigma}}{\varrho_m} - \frac{\varrho}{\varrho_m}\frac{\dot{G}_4}{G_4}. \quad (29)$$

So, Eq. (27) takes the form:

$$\dot{\epsilon}_{\delta\sigma} = \epsilon_{\delta\sigma} \left[ 3\frac{\dot{a}}{a}(1+w) + 2\frac{\dot{c}}{c}\frac{1+\epsilon_{\delta\sigma}}{1+\epsilon} + \left( \frac{\dot{G}_4}{G_4} - 4\frac{\dot{c}}{c} \right) (1+\epsilon_{\delta\sigma}) \right]. \quad (30)$$

Note,  $G_4$  is given by Eq. (13). We also assume that  $\dot{G}_5 \neq 0$ . Then, we obtain

$$\frac{\dot{G}_4}{G_4} = -4\frac{\dot{c}}{c} + 2\frac{\dot{G}_5}{G_5} + \frac{16\pi G_5 \sigma_0 \Omega_0^2}{3c^4(1-\Omega_0^2)}\frac{\dot{c}}{c} - \frac{4\pi G_5 \sigma_0 \Omega_0^2}{3c^4(1-\Omega_0^2)}\frac{\dot{G}_5}{G_5}. \quad (31)$$

As before, the first term on the RHS of Eq. (30) can be neglected, if we assume  $|\dot{c}/c|, |\dot{G}_5/G_5| \gg \dot{a}/a$ . The coefficients of  $\dot{c}/c$  and  $\dot{G}_5/G_5$  for the last two terms on the RHS of Eq. (31) are of the order  $10^{-15}$  with values of quantities assumed in the RS model [20, 21, 22], so the last two terms can be neglected compared to the first two terms. Then, Eq. (30) becomes

$$\dot{\epsilon}_{\delta\sigma} \approx \epsilon_{\delta\sigma} \left( 2\frac{\dot{G}_5}{G_5} - \frac{6+8\epsilon\dot{c}}{1+\epsilon}\frac{\dot{c}}{c} \right) (1+\epsilon_{\delta\sigma}). \quad (32)$$

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<sup>3</sup>Indeed, in the equations under consideration, i.e. Eqs. (11,12,14), the fine-tuned values for the brane tensions  $\sigma_{0,1/2}$  are taken into account, and  $\varrho$  and  $\wp$  in these equations do not include contributions from  $\sigma_{0,1/2}$ .

First, we see from Eq. (30) that corrections to the fine-tuned brane tensions (10) will grow fast as the brane universe expands, if we assume  $\dot{c} = 0 = \dot{G}_5$ . Therefore, in the brane world cosmologies the corrections to the brane tensions at an early time have to be extremely small or there has to exist some mechanism that suppresses growth of corrections to the brane tensions, in order to avoid contradiction with small cosmological constant observed in our present universe. Second, from Eq. (32) we see that, unlike the case of the VSL standard cosmology, assuming  $\dot{c} < 0$  with the constant (five-dimensional) Newton's constant is not enough for resolving the cosmological constant problem. In fact, the fast decreasing  $c$  will make the corrections to the brane tensions grow extremely fast, leading to very large value of cosmological constant in brane universe. In order to resolve the cosmological constant problem, we have to additionally assume that  $G_5$  decreases faster than  $c$ :

$$2\frac{\dot{G}_5}{G_5} < \frac{6 + 8\epsilon}{1 + \epsilon} \frac{\dot{c}}{c}. \quad (33)$$

When this condition is satisfied, the correction  $\delta\sigma = \delta\sigma_0 + \Omega_0^4 \delta\sigma_{1/2}$  to the fine-tuned brane tensions  $\sigma_{0,1/2}$  is converted into the conventional matter while  $c$  and  $G_5$  decrease, thereby the brane tensions are pushed back to the fine-tuned values (10).

### 2.3 Gradually changing speed of light

In this subsection, we examine the case in which  $c$  varies as a power law in the scale factor  $a$ , as was first considered in Ref. [6]. Since the issue on the flatness problem is along the same line as the standard cosmology case, we concentrate on the cosmological constant problem. We can rewrite Eq. (26) with  $\varrho_m = w\varphi_m c^2$  in the following form:

$$\left(G_4 \varrho_m a^{3(w+1)}\right)' = G_4 \varrho_{\delta\sigma} a^{3(w+1)} \left(2\frac{\dot{c}}{c} - \frac{\dot{G}_4}{G_4}\right) + \frac{3kc\dot{c}a^{3w+1}}{4\pi}. \quad (34)$$

By applying the same approximation used in the previous subsection, namely that the last two terms on the RHS of Eq. (31) can be neglected and similarly  $\Omega_0 \approx 0$  in Eq. (13), i.e.  $G_4 \approx m_0 G_5$ , we can approximate Eq. (34) to the following form:

$$\left(G_4 \varrho_m a^{3(w+1)}\right)' \approx \frac{8\pi\sigma_0\delta\sigma G_5^2 a^{3(w+1)}}{3b_0 c^6} \left(3\frac{\dot{c}}{c} - \frac{\dot{G}_5}{G_5}\right) + \frac{3kc\dot{c}a^{3w+1}}{4\pi}. \quad (35)$$

Assuming that  $c$  and  $G_5$  vary gradually like  $c = c_0 a^n$  and  $G_5 = \tilde{G}_5 a^q$  with constant  $c_0$ ,  $\tilde{G}_5$ ,  $n$  and  $q$ , we can integrate Eq. (35) to obtain

$$G_4 \varrho_m a^{3(w+1)} = \frac{3n - q}{3w + 2q - 6n + 3} \frac{8\pi\tilde{G}_5^2 \sigma_0 \delta\sigma}{3b_0 c_0^6} a^{3w+2q-6n+3}$$

$$+\frac{3nk_0^2}{4\pi(3w+2n+1)}a^{3w+2n+1}+B, \quad (36)$$

where  $B$  is an integration constant. Substituting this into Eq. (11) with  $\varrho = \varrho_m + \varrho_{\delta\sigma}$ , we have

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} \frac{B}{a^{3w+3}} + \frac{32(w+1)\pi^2}{3(3w+2q-6n+3)} \frac{\tilde{G}_5^2 \sigma_0 \delta\sigma}{b_0 c_0^6} a^{2q-6n} - \frac{3w+2n-1}{3w+2n+1} k c_0^2 a^{2n-2}, \quad (37)$$

where the second and the third terms on the RHS respectively correspond to the cosmological constant and the curvature terms. The ratio of the matter energy density and the corrections to the brane tensions is given by

$$\frac{\varrho_m}{\varrho_{\delta\sigma}} = \frac{2b_0 c_0^6 B}{\tilde{G}_5^2 \sigma_0 \delta\sigma} a^{-3w+6n-2q-3} + \frac{9nk_0^8 b_0}{16\pi^2 \tilde{G}_5^2 \sigma_0 \delta\sigma (3w+2n+1)} a^{8n-2q+1} + \frac{6n-2q}{3w-6n+2q+3}. \quad (38)$$

Note, in the case of the VSL brane world cosmology, the cosmological constant term in the Friedmann equation behaves as  $\sim a^{2q-6n}$  with the *negative* coefficient for the  $n$  term in the index, whereas that of the VSL standard cosmology behaves as  $\sim a^{2n}$  [6, 8]. So, we have to further assume that the Newton's constant  $G_5$  gradually decreases fast enough in order for the  $B$  term dominates the cosmological constant term as  $a$  increases. The condition for this to happen is given by

$$3w-6n+2q-3 < 0. \quad (39)$$

As in the VSL standard cosmology case [6, 8], the condition for the curvature term to be dominated by the  $B$  term as  $a$  increases is given by

$$3w+2n+1 < 0. \quad (40)$$

So, unlike the VSL standard cosmology case, resolving the flatness problem does not necessary guarantees resolving the cosmological constant problem.

Next, we consider the quasi cosmological constant problem. The quasi cosmological constant problem stems from the fact that the cosmological constant observed in our present universe is not exactly zero but has a small positive value. One of the appealing features of the VSL models is that it resolves not only the cosmological constant problem but also the quasi cosmological constant problem. As can be seen in Eq. (38), for a suitable choice of  $n$  and  $q$  the ratio of the matter density and the corrections to the brane tensions (therefore the vacuum energy density) approaches a constant value  $(6n-2q)/(3w-6n+2q+3)$  as  $a$  increases. When Eq. (39) is satisfied, just like the VSL standard cosmology case, there is no solution to the quasi cosmological constant problem, since the ratio (38) approaches infinity (i.e. the cosmological constant approaches zero) as  $a$  increases. However, unlike the VSL standard cosmology case, we cannot

resolve the quasi cosmological constant problem just by assuming  $3w - 6n + 2q - 3 > 0$ : We have to further assume that  $8n - 2q + 1 < 0$  so that the second term on the RHS of Eq. (38) approaches zero as  $a$  increases. Also, in order for the ratio to approach a *positive* value, we have to additionally assume that  $3n > q$ . Note, the condition  $3n > q$  also corresponds to the condition that the cosmological constant term on the RHS of Eq. (37) approaches zero as  $a$  increases. When these conditions are satisfied, the corrections  $\delta\sigma_{0,1/2}$  to the fine-tuned brane tensions will reach some equilibrium values which give rise to a small positive cosmological constant in the brane universe, as  $a$  increases.

### 3 The VSL Cosmology in the RS2 Model

In this section, we consider the VSL cosmology in the RS model with noncompact extra dimension. In the preferred frame in which the principle of minimal coupling holds, the action takes the form:

$$S = \int d^5x \left[ \sqrt{-\hat{g}} \left( \frac{\psi}{16\pi G_5} \mathcal{R} - \Lambda \right) + \mathcal{L}_\psi \right] + \int d^4x \sqrt{-g} [\mathcal{L}_{mat} - \sigma], \quad (41)$$

where  $\mathcal{L}_{mat}$  is the Lagrangian density for matter fields confined on the brane with tension  $\sigma$  at  $y = 0$ . So, the energy-momentum tensor is given by

$$\mathcal{T}_{MN} = -\hat{g}_{MN}\Lambda + \delta_M^\mu \delta_N^\nu \left( \mathcal{T}_{\mu\nu}^{mat} - g_{\mu\nu}\sigma \right) \frac{\delta(y)}{b}, \quad (42)$$

with the energy-momentum tensor  $\mathcal{T}_{\mu\nu}^{mat} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_{mat})}{\delta g^{\mu\nu}}$  for the brane matter fields having the usual perfect fluid form:

$$\mathcal{T}^{mat\mu}_{\nu} = \text{diag}(-\varrho c^2, \wp, \wp, \wp). \quad (43)$$

The Einstein's equations (4) therefore take the form:

$$\frac{3}{c^2 n^2} \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) - \frac{3}{b^2} \left[ \frac{a'}{a} \left( \frac{a'}{a} - \frac{b'}{b} \right) + \frac{a''}{a} \right] + \frac{3k}{a^2} = \frac{8\pi G_5}{c^4} \left[ \Lambda + (\sigma + \varrho c^2) \frac{\delta(y)}{b} \right], \quad (44)$$

$$\begin{aligned} & \frac{1}{b^2} \left[ \frac{a'}{a} \left( 2\frac{n'}{n} + \frac{a'}{a} \right) - \frac{b'}{b} \left( \frac{n'}{n} + 2\frac{a'}{a} \right) + 2\frac{a''}{a} + \frac{n''}{n} \right] + \\ & \frac{1}{c^2 n^2} \left[ \frac{\dot{a}}{a} \left( 2\frac{\dot{n}}{n} - \frac{\dot{a}}{a} \right) + \frac{\dot{b}}{b} \left( \frac{\dot{n}}{n} - 2\frac{\dot{a}}{a} \right) - 2\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} \right] - \frac{k}{a^2} = -\frac{8\pi G_5}{c^4} \left[ \Lambda + (\sigma - \wp) \frac{\delta(y)}{b} \right], \end{aligned} \quad (45)$$

$$\frac{n'}{n} \frac{\dot{a}}{a} + \frac{a'}{a} \frac{\dot{b}}{b} - \frac{\dot{a}'}{a} = 0, \quad (46)$$

$$\frac{3}{b^2} \frac{a'}{a} \left( \frac{a'}{a} + \frac{n'}{n} \right) - \frac{3}{c^2 n^2} \left[ \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) + \frac{\ddot{a}}{a} \right] - \frac{3k}{a^2} = -\frac{8\pi G_5}{c^4} \Lambda. \quad (47)$$

The effective four-dimensional equations of motion on the three-brane worldvolume can be obtained [42] by taking the jumps and the mean values of the above Einstein's equations across  $y = 0$  and then applying the boundary conditions on the first derivatives of the metric components due to the  $\delta$ -function singularity on the brane. We consider the solution invariant under the  $\mathbf{Z}_2$ -symmetry  $y \rightarrow -y$ . We define  $t$  to be the cosmic time for the brane universe, i.e.  $n(t, 0) = 1$ . We also define the  $y$ -coordinate to be proportional to the proper distance along the  $y$ -direction with  $b$  being the constant of proportionality, namely  $b' = 0$ . We further assume that the radius of the extra space is stabilized, i.e.  $\dot{b} = 0$ . Making use of these assumptions, we define the  $y$ -coordinate such that  $b = 1$ . The resulting effective Friedmann equations have the forms:

$$\left( \frac{\dot{a}_0}{a_0} \right)^2 = \frac{32\pi^2 G_5^2}{9c^6} (\varrho^2 c^4 + 2\sigma \varrho c^2) + \frac{c^2 a''_{R0}}{a_0} + \frac{8\pi G_5}{3c^2} \left( \Lambda + \frac{4\pi G_5}{3c^4} \sigma^2 \right) - \frac{kc^2}{a_0^2}, \quad (48)$$

$$\frac{\ddot{a}_0}{a_0} = -\frac{32\pi^2 G_5^2}{9c^6} (2\varrho^2 c^4 + \sigma \varrho c^2 + 3\sigma \wp + 3\varrho \wp c^2) - \frac{c^2 a''_{R0}}{a_0} + \frac{32\pi^2 G_5^2}{9c^6} \sigma^2, \quad (49)$$

where  $a''_R$  denotes the regular part of  $a''$  and the subscript 0 denotes the quantities evaluated at  $y = 0$ , e.g.  $a_0(t) \equiv a(t, 0)$ .

The  $a''_{R0}$ -terms (called “dark radiation” terms) in the above Friedmann equations originate from the Weyl tensor of the bulk and thus describe the backreaction of the bulk gravitational degrees of freedom on the brane [42, 43, 44, 45, 46]. These terms can be evaluated by solving  $a$  as a function of  $y$  from the following equation obtained from the diagonal component Einstein's equations:

$$3 \frac{a''_R}{a} + \frac{n''_R}{n} = -\frac{16\pi G_5}{3c^4} \Lambda, \quad (50)$$

supplemented by the following relation obtained from the (0, 4)-component Einstein's equation with the assumed  $\dot{b} = 0$  condition:

$$n(t, y) = \lambda(t) \dot{a}(t, y), \quad (51)$$

where  $\lambda(t)$  is an arbitrary function of  $t$ . The resulting expression is

$$a''_R = \frac{C}{a^3} - \frac{4\pi G_5}{3c^4} \Lambda a - \frac{4\pi G_5}{3a^3} \Lambda \int c^{-5} \dot{c} a^4 dt, \quad (52)$$

where  $C$  is an integration constant. For the case where  $c$  suddenly jumps from  $c_-$  to  $c_+$  at a critical time  $t_c$ , the integrand in Eq. (52) becomes a  $\delta$ -function  $c^{-5} \dot{c} = \frac{c_+^4 - c_-^4}{4c_+^4 c_-^4} \delta(t - t_c)$ , so the integral term can be easily evaluated<sup>4</sup>.

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<sup>4</sup>For the  $c(t) = c_0 a^n$  case, the integral can also be easily integrated to be  $\int c^{-5} \dot{c} a^4 dt = -\frac{nc_0^{-4}}{4(n-1)} a^{-4(n-1)}$ .

We assume that  $\sigma$  is fine-tuned as in Eq. (10), and  $\sigma \gg \varrho c^2, \wp$  [47, 48]. To the leading order, the Friedmann equations (48,49) then take the following forms:

$$\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{8\pi G_4}{3}\varrho + \frac{Cc^2}{a_0^4} - \frac{kc^2}{a_0^2}, \quad (53)$$

$$\frac{\ddot{a}_0}{a_0} = -\frac{4\pi G_4}{3}\left(\varrho + 3\frac{\wp}{c^2}\right) - \frac{Cc^2}{a_0^4}, \quad (54)$$

where the effective four-dimensional Newton's constant is given by

$$G_4 = \frac{8\pi G_5^2 \sigma}{3c^4}, \quad (55)$$

and we have absorbed the coefficient of the last term in Eq. (52) evaluated at  $y = 0$  into the integration constant  $C$ . From these effective Friedmann equations, we obtain the following generalized energy conservation equation:

$$\dot{\varrho} + 3\frac{\dot{a}_0}{a_0}\left(\varrho + \frac{\wp}{c^2}\right) = -\frac{\dot{G}_4}{G_4}\varrho + \frac{3kc\dot{c}}{4\pi G_4 a_0^2} - \frac{3Cc\dot{c}}{4\pi G_4 a_0^4}. \quad (56)$$

So, in the case of VSL cosmology in the RS2 model, the dark radiation terms make additional contribution to nonconservation of mass density in the brane universe. The constant  $C$  can take an arbitrary value, depending on the geometry in the bulk, but its upper limit can be constrained by nucleosynthesis [43]. In Ref. [49], it is shown by considering the brane world cosmology on a domain wall moving in the bulk of the AdS Schwarzschild space that  $C$  can be identified as the mass of the AdS black hole. The requirement of existence of regular horizon for the bulk spacetime gives rise to the lower bound for  $C$ .

### 3.1 The flatness problem

In this subsection, we examine the condition under which the flatness problem can be resolved in the VSL cosmology of the RS2 model. From Eq. (53) we see that the critical density is now given by

$$\varrho_c = \frac{3}{8\pi G_4} \left[ \left(\frac{\dot{a}_0}{a_0}\right)^2 - \frac{Cc^2}{a_0^4} \right]. \quad (57)$$

So, for a sufficiently large value of  $C$ , the critical mass density becomes negative, in which case the flatness problem gets worse as we will see. The time derivative of  $\epsilon = \varrho/\varrho_c - 1$  is still given by Eq. (17). However, the dark radiation terms modify  $\dot{\varrho}/\varrho$  and  $\dot{\varrho}_c/\varrho_c$  in the following way

$$\frac{\dot{\varrho}}{\varrho} = -3\frac{\dot{a}_0}{a_0}(1+w) - \frac{\dot{G}_4}{G_4} + 2\frac{\dot{c}}{c} \left[ \frac{\epsilon}{1+\epsilon} - \frac{3Cc^2}{8\pi G_4 a_0^4 \varrho} \right],$$

$$\frac{\dot{\varrho}_c}{\varrho_c} = -\frac{\dot{a}_0}{a_0}[2 + (1 + \epsilon)(1 + 3w)] - \frac{\dot{G}_4}{G_4} - \frac{3Cc\dot{c}}{4\pi G_4 a_0^4 \varrho_c}. \quad (58)$$

Substituting these into Eq. (17), we obtain

$$\begin{aligned} \dot{\epsilon} &= (1 + \epsilon)\epsilon \frac{\dot{a}_0}{a_0}(1 + 3w) + 2\frac{\dot{c}}{c}\epsilon + \frac{3Cc\dot{c}}{4\pi G_4 a_0^4 \varrho_c}\epsilon \\ &= (1 + \epsilon)\epsilon \frac{\dot{a}_0}{a_0}(1 + 3w) + \frac{2\left(\frac{\dot{a}_0}{a_0}\right)^2}{\left(\frac{\dot{a}_0}{a_0}\right)^2 - \frac{Cc^2}{a_0^4}}\frac{\dot{c}}{c}\epsilon, \end{aligned} \quad (59)$$

where Eq. (57) is used in the second equality. So, when  $C < 0$ , the flatness problem can be resolved by assuming rapid enough decreasing  $c$ , just as in the RS1 case. However, when  $C \geq 0$ , the flatness problem can be resolved by assuming  $|\dot{c}/c| \gg \dot{a}_0/a_0$ , provided  $C$  satisfies  $C < (a_0\dot{a}_0/c)^2$ . Too large positive value for  $C$ , leading to negative  $\varrho_c$ , will drive  $\epsilon$  away from zero fast. This upper limit on  $C$  is compatible with the upper limit imposed by the nucleosynthesis constraint [43] and the fact that  $C$  should not be large in order not to give a large contribution to the current expansion of the universe.

### 3.2 The cosmological constant problem

We now examine the condition under which the cosmological constant problem can be resolved in the VSL cosmology of the RS2 model. Separating the contribution to the mass density into that from the ordinary matter and that from the correction to the brane tension, i.e.  $\varrho = \varrho_m + \varrho_{\delta\sigma}$ , we can rewrite the generalized conservation equation (56) in the following form:

$$\dot{\varrho}_m + 3\frac{\dot{a}_0}{a_0}\left(\varrho_m + \frac{\wp_m}{c^2}\right) = -\dot{\varrho}_{\delta\sigma} - \frac{\dot{G}_4}{G_4}\varrho + \frac{3kc\dot{c}}{4\pi G_4 a_0^2} - \frac{3Cc\dot{c}}{4\pi G_4 a_0^4}. \quad (60)$$

$\dot{\varrho}_{\delta\sigma}/\varrho_{\delta\sigma}$  has the same form (28) as the RS1 case, but  $\dot{\varrho}_m/\varrho_m$  receives an additional contribution from the dark radiation terms:

$$\frac{\dot{\varrho}_m}{\varrho_m} = -3\frac{\dot{a}}{a}(1 + w) - 2\frac{\dot{c}}{c}\frac{\varrho_c}{\varrho_m} + 2\frac{\dot{c}}{c}\frac{\varrho + \varrho_{\delta\sigma}}{\varrho_m} - \frac{\varrho}{\varrho_m}\frac{\dot{G}_4}{G_4} - \frac{3Cc\dot{c}}{4\pi G_4 a_0^4 \varrho_m}. \quad (61)$$

Eq. (27) therefore takes the form:

$$\dot{\epsilon}_{\delta\sigma} = \epsilon_{\delta\sigma} \left[ 3\frac{\dot{a}}{a}(1 + w) + 2\frac{\dot{c}}{c}\frac{1 + \epsilon_{\delta\sigma}}{1 + \epsilon} + \left(\frac{\dot{G}_4}{G_4} - 4\frac{\dot{c}}{c}\right)(1 + \epsilon_{\delta\sigma}) + \frac{3Cc\dot{c}}{4\pi G_4 a_0^4 \varrho_m} \right]. \quad (62)$$

Making use of the expression for  $G_4$  in Eq. (55) and assuming  $\dot{G}_5 \neq 0$ , we can rewrite this as

$$\dot{\epsilon}_{\delta\sigma} = \epsilon_{\delta\sigma} \left[ 3\frac{\dot{a}}{a}(1 + w) + \left\{ 2\frac{\dot{G}_5}{G_5} - \frac{6 + 8\epsilon}{1 + \epsilon}\frac{\dot{c}}{c} \right\} (1 + \epsilon_{\delta\sigma}) + \frac{9Cc^6}{32\pi^2 G_5^2 \sigma a_0^4 \varrho_m} \frac{\dot{c}}{c} \right]. \quad (63)$$

So, in the brane world cosmology of the RS2 model with  $\dot{c} = 0$  and  $\dot{G}_5 = 0$ , the correction to the fine-tuned brane tension will grow fast as the brane universe expands, unless the correction is extremely small. Unlike the RS1 case considered in the previous section, there is a possibility of solving the cosmological constant without assuming  $\dot{G}_5 \neq 0$ . Namely, when  $\dot{G}_5 = 0$ , the dark radiation term plays a crucial role: The integration constant  $C$  has to take large enough positive value to be able to suppress the growth of  $\epsilon_{\delta\sigma}$  with time. Otherwise, we have to further assume rapid enough decreasing  $\dot{G}_5$ . Note, however, that too large positive value for  $C$  will drive  $\epsilon$  away from zero fast, as seen in the previous subsection. So, when  $\dot{G}_5 = 0$ ,  $C$  should not take too large or too small positive value in order to solve both flatness and cosmological constant problems.

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